

The problem of the quasisteady diffusion-controlled evaporation of a droplet moving in a viscous medium is solved numerically. The relative values of the droplet radius and velocity are determined, along with the total droplet evaporation time and the mass-transfer characteristics of the droplet and the medium.

The problem of the quasisteady evaporation of a spherical droplet in the presence of diffusion, where the rate of evaporation is limited by vapor mass transfer, was first investigated for evaporation in a nonmoving medium by Maxwell, who derived the following equation for the variation of the droplet radius R as a function of the evaporation time t [1]:

$$R^2 = R_0^2 - \frac{2D}{\rho_1} (C_s - C_\infty). \quad (1)$$

It is assumed that the temperature of the droplet remains constant and the saturated vapor concentration C_s obeys the condition $C_s \ll \rho_1$, where ρ_1 is the density of the droplet liquid.

Several papers ([2, 3] and others) analyze the evaporation problem for a droplet at rest relative to the external medium in a more general setting with regard for the presence of Stefan flow, the unsteadiness of the process due to heating of the droplet, and the variable physical properties of the droplet and the medium.

The motion of the evaporating droplet as a whole relative to the external medium exerts a significant influence on the evaporation process, but the problem then becomes much more complex to solve and must be analyzed under certain simplifying assumptions. The fundamental assumption is that the motion of the droplet is quasisteady, i.e., the resistance of the medium does not depend on the acceleration of the droplet. Moreover, it has been shown [4] that the reactive force due to the nonuniform transport of vapor from the surface of the droplet can be neglected and, given the assumption that $p_0/p \ll 1$, where p_0 is the partial pressure of the vapor on the droplet surface and p is the total pressure of the medium, Stefan flow can be neglected [1].

Along with these assumptions, we assume in the present study that the physical properties of the droplet liquid and the external medium are constant and the motion is axisymmetrical. The vapor concentration on the surface of the droplet is considered to be constant and equal to the saturation concentration C_s , and the temperature of the droplet and the medium is constant.

We solve the problem by an approximation method based on the quasisteady approach. The time axis $0 \leq t < \infty$ is partitioned into intervals $\delta_1, \dots, \delta_n, \dots$ of constant length Δt , and the problem of mass transfer from a droplet moving with a constant velocity is investigated in each interval. The problem is solved in dimensionless variables. The dimensionless concentration is $C = (C' - C_\infty)/(C_s - C_\infty)$ (C' is the mass concentration of the vapor). The characteristic scales for the velocity and length in the interval δ_1 are U_0 and R_0 (the initial values of the droplet velocity and radius), and in δ_{n+1} they are U_n and R_n (the velocity and radius determined by the solution of the problem in the preceding interval). It is assumed that the liquid of the droplet and the external medium are Newtonian fluids. Because of the axial symmetry of the problem we can introduce the stream function ψ and vorticity ω to describe the flow. In a spherical coordinate system (r, θ, φ) with origin at the center of gravity of the droplet and polar axis $\theta = 0$ directed counter to the direction of motion of the droplet the Navier-Stokes equations in ψ and ω have the form

$$\sin \theta \left[\frac{\partial \psi_i}{\partial \theta} \frac{\partial}{\partial r} \left(\frac{\omega_i}{r \sin \theta} \right) - \frac{\partial \psi_i}{\partial r} \frac{\partial}{\partial \theta} \left(\frac{\omega_i}{r \sin \theta} \right) \right] = \frac{2}{Re_i^n} E^2(\omega_i r \sin \theta), \quad (2)$$

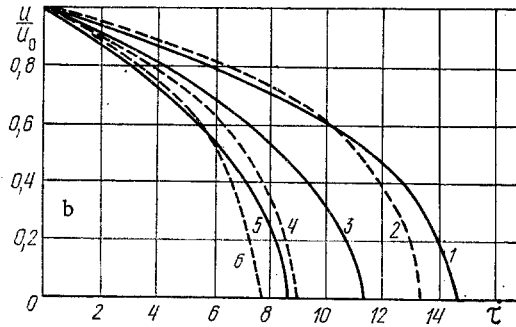
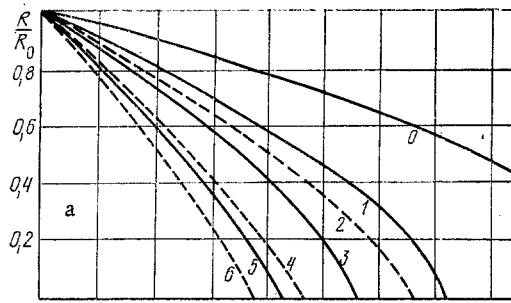


Fig. 1

Fig. 1. Dimensionless droplet radius (a) and dimensionless relative droplet velocity (b) versus time. 1) $Sc = 2$; 2) 0.4; 3) 8; 4) 1.6; 5) 10; 6) 2.4.

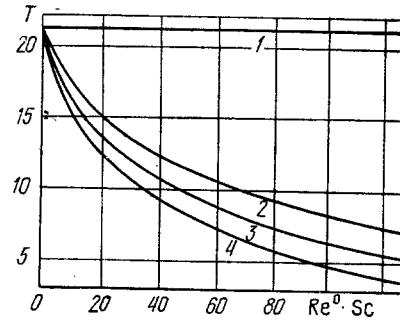


Fig. 2

Fig. 2. Total evaporation time of droplet versus $Re^0 \cdot Sc$. 1) $Re^0 = 0$; 2) 10; 3) 50; 4) 100.

$$E^2(\psi_i) + \omega_{ir} \sin \theta = 0. \quad (3)$$

The index i takes the values $i = 1$ for the droplet and $i = 2$ for the external medium;

$$E^2 = \frac{\partial^2}{\partial r^2} + \frac{\sin \theta}{r^2} \frac{\partial}{\partial \theta} \left(\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \right);$$

and $Re_i^n = 2R_n U_n / \nu_i$ denotes the Reynolds numbers.

We augment Eqs. (2) and (3) with the conditions of nondisturbance of the flow far from the droplet, boundedness of the flow at the center of the droplet, as well as impermeability of the droplet surface and continuity of the velocity and tangential component of the stress tensor at that surface.

The parameters governing the behavior and structure of the flow are the external Reynolds number Re_2^n and the ratios between the dynamic viscosities μ and the densities ρ inside and outside the droplet.

After solving the Navier–Stokes equations with the appropriate boundary and endpoint matching conditions, in the same time interval we solve the steady mass-transfer problem for the vapor in the external medium, as described by the equation

$$Sc \frac{Re_2^n}{2} \left(\frac{\partial \psi_2}{\partial \theta} \frac{\partial C}{\partial r} - \frac{\partial \psi_2}{\partial r} \frac{\partial C}{\partial \theta} \right) = r^2 \sin \theta \Delta C \quad (4)$$

and the boundary conditions

$$C_{r=1} = 1, \quad C_{r \rightarrow \infty} \rightarrow 0 \quad (5)$$

($Sc = \nu_2 / D$ is the Schmidt number). The solution of problem (4), (5) yields the Sherwood number:

$$Sh^n = \int_S \frac{\partial C}{\partial r} dS.$$

For quasisteady evaporation limited by vapor mass transfer the droplet radius obeys the differential equation

$$\frac{dR}{dt} = - \frac{C_s - C_\infty}{\rho_1} \frac{D}{2R} \text{Sh}. \quad (6)$$

Assuming that $R = R_n$ and $\text{Sh} = \text{Sh}^n$ in the interval δ_n and integrating (6) in that interval, we obtain

$$R_{n+1} = R_n - \frac{C_s - C_\infty}{2\rho_1} \frac{D}{R_n} \text{Sh}^n \Delta t$$

or, in dimensionless form,

$$\frac{R_{n+1}}{R_n} = 1 - \frac{C_s - C_\infty}{2\rho_1} \text{Sh}^n \tau, \quad (7)$$

where $\tau = \Delta t D / R_n^2$.

The parameter Re_2^{n+1} , which corresponds to the new value of the radius R_{n+1} , is determined by solving the equation of motion of the droplet as a whole, which has the following form without regard for the reactive force:

$$\frac{d}{dt}(m\bar{V}) = \bar{P} + \bar{F}, \quad (8)$$

where \bar{V} is the absolute velocity, \bar{F} is the resistive force of the medium $F = C_x \pi R^2 \rho_2 U^2 / 2$; U is the relative velocity of the droplet, and \bar{P} is the force of gravity.

It is assumed in the quasisteady approximation that the resistive force equalizes the gravity force in each interval δ_n . Consequently,

$$1/2 \pi \rho_2 C_x R^2 U^2 = 4/3 \pi \rho_1 R^3 g. \quad (9)$$

From (9) we obtain

$$C_x \text{Re}_2^2 = 32\rho \frac{g}{3v_2^2} R^3. \quad (10)$$

Equation (10) is solved for Re_2 by successive approximations. The initial approximation is calculated on the basis of the approximation formula proposed for the coefficient of fluid friction in [5], by numerical solution of the equation obtained from (10) in this case for Re_2 . The subsequent approximations are determined from the expression

$$\text{Re}_2^{k+1} = \left(32\rho \frac{g}{3v_2^2} R_n^3 \frac{1}{C_x^k} \right)^{1/2}, \quad k = 0, 1, \dots,$$

in which C_x^k is the value of the friction coefficient obtained by solution of the hydrodynamical problem for $\text{Re}_2 = \text{Re}_2^k$.

We then determine the value of U_{n+1} and repeat the process of solving the Navier–Stokes equation with appropriate boundary and matching conditions, as well as problem (4), (5) for determining the new droplet radius from (7), and so on until the value of the radius is smaller than a prescribed number ϵ .

The Navier–Stokes and mass-transfer equations are solved by the method of finite differences with the use of alternating-direction implicit schemes similar to the schemes used in [6, 7].

The solution of the problem described above yields the Sherwood number, the dimensionless radius, and the dimensionless velocity of the droplet as a function of the time and the total evaporation time of the droplet for various values of the governing parameters of the problem: Re_2 , μ , ρ , Sc , $\kappa = (C_s - C_\infty) / \rho_1$. It has been shown [5] that variations of the parameter Re_1 over a wide range for fixed values of Re_2 and μ do not significantly affect the flow outside the droplet, and so it is assumed in the calculations that $\text{Re}_1 = \text{Re}_2 = \text{Re}$.

It follows from expression (7) that if the time dependences of the droplet radius and velocity are known for a certain value of the parameter $\kappa = \kappa^*$, then the dependence for other values of the parameter can be approximated by curves obtained from those corresponding to κ^* by a change of scale along the τ axis. The scale factor is equal to the ratio κ / κ^* . In this investigation the main calculations are carried out for $\kappa^* = 10^{-2}$.

Inasmuch as we are investigating the problem in the quasisteady regime, in place of the time dependence of the Sherwood number it is practical to analyze the dependence of this number on the Reynolds and Schmidt numbers (for fixed values of all the other parameters). The values obtained for the Sherwood number by numerical solution of the problem are approximated by the expression

$$\text{Sh} = 2 + K \frac{(\text{Re Sc})^{7/6}}{(1 + 0.4 \text{Re}^{2/3})(1 + 5\text{Sc}^{5/6})} \quad (11)$$

The degree of intensification of mass transfer due to the relative motion of the droplet can be characterized by the intensity factor Q , which is defined as the ratio Sh/Sh_0 , where Sh_0 is the Sherwood number for mass transfer of the stationary droplet. We then obtain from (11)

$$Q = 1 + \frac{K}{2} \frac{(\text{Re Sc})^{7/6}}{(1 + 0.4\text{Re}^{2/3})(1 + 5\text{Sc}^{5/6})}$$

The intensity factor for the droplet evaporation process decreases from the initial maximum value, which is determined by the parameters Re^0 and Sc , to unity ($Q=1$ corresponds to evaporation of the droplet at rest).

The errors of the values of Sh calculated according to (11) for $K=1.84$ (for $\mu=1$) amount to not more than 5-7% relative to the values of Sh obtained by numerical solution of the problem, in the range of parameters where the numerical calculations are performed ($\text{Re} \leq 100$, $\text{Sc} \leq 10$).

As $\text{Re}, \text{Sc} \rightarrow \infty$ expression (11) goes over to

$$\text{Sh} = 2 + K^0 \text{Re}^{1/2} \text{Sc}^{1/3}, \quad (12)$$

which is obtained by solving the external mass-transfer problem for the droplet by the methods of boundary-layer theory [1]. Expression (12) is also widely used for the processing of experimental data on the external mass transfer of a droplet [8]. The value $K^0 = 0.92$ corresponding to the given value $K=1.84$ agrees with the values of K^0 given in [8], which are taken from various authors.

The dependence of the Sherwood number on μ is approximated within 5% error limits by an expression proposed in [8]:

$$\text{Sh}(\mu) = (\text{Sh}(0) + \mu \text{Sh}(\infty))/(1 + \mu),$$

where $\text{Sh}(0)$ and $\text{Sh}(\infty)$ are the values of the Sherwood number for the mass transfer of a gas bubble and a solid plate, respectively.

Figure 1 gives the curves obtained for the dimensionless droplet radius R/R_0 and dimensionless velocity U/U_0 as a function of the evaporation time $\tau = tD/R_0^2$ for $\mu=1$, two values of the parameter Re^0 : $\text{Re}^0=10$ (solid curves) and $\text{Re}^0=50$ (dashed curves), and various values of the parameter Sc . Curve 0 in Fig. 1a is plotted for $\text{Re}^0=0$, which corresponds to the case of droplet evaporation in a nonmoving medium and coincides with the dependence described by expression (1).

The motion of the droplet relative to the external medium intensifies the mass-transfer process, inducing a substantial increase in the evaporation rate in comparison with the evaporation of the stationary droplet. Figure 2 gives the total droplet evaporation time T as a function of the product $\text{Re}^0 \cdot \text{Sc}$ for various values of Re^0 . Line 1 corresponds to the evaporation of the stationary droplet, where the evaporation time does not depend on the parameter Sc .

NOTATION

R , droplet radius; t , time; D , diffusion coefficient; ρ , density; C_S , mass concentration of saturated vapor; C_∞ , mass concentration of vapor far from the droplet; $C = (C' - C_\infty)/(C_S - C_\infty)$, dimensionless vapor concentration; (r, θ, φ) , spherical coordinates; $\text{Re} = 2RU/v$, Reynolds number; $\text{Sc} = v/D$, Schmidt number; Sh , Sherwood number; $\tau = tD/R^2$, dimensionless time; C_x , coefficient of fluid friction; m , mass of the droplet; T , total droplet evaporation time; $\kappa = (C_S - C_\infty)/\rho_1$; μ , ratio of dynamic viscosities inside and outside the droplet; Q , mass-transfer intensity factor.

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MATHEMATICAL MODELING OF A TRANSIENT HEAT-CONDUCTION PROCESS

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A numerical algorithm is proposed for solution of the transient heat-conduction equation by the Monte Carlo method. The calculated values of the temperature are compared with experimental data.

Let D be a finite domain of space (x, y, z) with boundaries Γ , $\bar{D} = D + \Gamma$. In the cylinder $\bar{Q}_T = \bar{D} \times [0 \leq t \leq T]$ with lateral surface $\Omega = \Gamma \times [0, T]$ it is required to find a solution $U(x, y, z, t)$ of the problem

$$\frac{\partial U}{\partial t} = a \left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} \right) + \varphi(x, y, z, t), \quad (x, y, z) \in D, \quad t > 0,$$

$$U(x, y, z, 0) = g(x, y, z), \quad (x, y, z) \in \bar{D},$$

$$U|_{\Omega} = f(x, y, z, t), \quad (x, y, z) \in \Gamma, \quad 0 \leq t \leq T, \quad (1)$$

in which φ , f , and g are continuous functions and $f(x, y, z, 0) = g(x, y, z)$ on Γ .

In the space (x, y, z) we introduce a uniform grid $(x_m, y_n, z_l) = (mh, nh, lh)$ with mesh spacing h , representing the set of points of intersection of the planes $x = mh$, $y = nh$, $z = lh$, where m, n, l are integers. Let ω_h be the set of interior nodes of the grid, i.e., nodes belonging to domain D , and let γ_h be the set of boundary nodes, i.e., nodes belonging to Γ or lying outside the domain \bar{D} and situated at a distance from Γ smaller than the mesh spacing. Let $\bar{\omega}_h = \omega_h + \gamma_h$.

We introduce a grid with respect to the variable t : $\bar{\omega}_\tau = \{t_i = i\tau, i = \overline{0, k+1}\}$, where τ is the mesh spacing and $k = [T/\tau]$ is the integer part of the number T/τ . Let $\omega_\tau = \{t_i = i\tau, i = \overline{1, k}\}$. We denote $\gamma_{h\tau} = \gamma_h \times \omega_\tau$, $\bar{\omega}_{h\tau} = \bar{\omega}_h \times \omega_\tau$, $\omega_{h\tau} = \omega_h \times \omega_\tau$. The set of nodes of the grid $\bar{\omega}_{h\tau}$ situated in the hyperplane $t = t_i$ is called a layer.

Approximating the initial heat-conduction equation by an implicit computing grid and putting

$$\tau a/h^2 = 1, \quad (2)$$

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